

Role of cracks in the creep deformation of brittle polycrystalline ceramics

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A discussion is presented of the effect of cracks formed during the creep deformation of polycrystalline ceramics on the rate of creep deformation. Four distinct regions of the creep curve are identified. In region I, creep is controlled by the basic creep mechanism without cracks. Creep in region II is the combined effect of elastic creep by crack growth due to time-dependent changes in elastic properties and crack-enhanced creep described by Weertman. In region III, in which the cracks have reached their final size, the rate of creep is governed by the sole effect of crack-enhanced creep. Region IV, not discussed in detail, is represented by accelerated creep due to crack coalescence prior to failure. The apparent stress exponent of crack-enhanced creep is shown to be governed by the value of the stress exponent of the basic creep mechanism as well as the stress dependence of the number of cracks formed per unit area or volume. The dependence of crack density on grain size also modifies the grain-size dependence of Nabarro–Herring and Coble creep. Depending on the specific mechanism of crack growth, region II creep can exhibit an apparent activation energy which can differ from the corresponding values for region I and III creep. Detailed microstructural information on crack size, crack density and other relevant variables is required for the quantitative analysis of the creep kinetics of polycrystalline ceramics subject to crack formation.

1. Introduction

The mechanisms of creep deformation of polycrystalline brittle ceramics by volume [1, 2] or grain-boundary diffusion [3] or dislocation motion [4, 5] are generally well accepted. Diffusional processes, dominant at low values of stress and temperature give rise to viscous (or linear) creep, for which the rate of creep depends linearly on the value of stress. In contrast, dislocation creep, generally dominant in brittle ceramics at high values of stress and temperature, exhibits non-linear behaviour for which the creep rate is proportional to the stress raised to an exponent, $n > 1$.

Extensive experimental observations of the creep kinetics for a wide variety of ceramic materials, however, indicate the existence of non-linear creep over ranges of stress and temperature at which dislocation-controlled creep behaviour should not be the dominant mechanism [6–11,

34]. In an earlier study such non-linear creep was attributed to non-linear grain-boundary sliding [12]. More recently, Blumenthal *et al.* [13] showed that for grain-boundary sliding accompanied by grain-boundary cracking, the stress exponent $n = 2$. Such non-linear creep is frequently observed in ceramics with a large grain size or with a residual pore phase [6, 9]. Crosby and Evans [14] suggested, therefore, that cracks play an active role in promoting non-linear creep in ceramic materials.

The purpose of this paper is to present a more quantitative discussion of the effect of cracks on the creep rate of brittle ceramics. At least two such effects exist, which can take place either separately or concurrently. The first effect, referred to here as crack-accelerated or crack-enhanced creep, treated theoretically by Weertman [15], results from the local stress field near the cracks and the associated transfer of stress to the

material adjacent to the cracks. The second effect arises from the growth of existing cracks, which leads to "elastic" creep due to time-dependent decrease in elastic moduli, recently proposed by the present authors [16]. Analyses of these two separate effects will be presented for two crack geometries. A discussion is then presented on how these effects influence the creep curve, the stress exponent of the creep rate, the apparent activation energy, the grain-size dependency and the stress state.

2. Analysis

2.1. General information

Weertman [15] derived expressions for crack-enhanced creep for a two-dimensional model consisting of a uniaxially stressed plate with through cracks. The present writers [17] obtained expressions for the rate of elastic creep of a three-dimensional, uni-axially stressed solid, with penny-shaped cracks. The expressions for these two creep mechanisms will not be re-derived, but restated only, without further discussion. Since crack-enhanced and elastic creep can occur simultaneously, expressions for the rate of elastic creep of a two-dimensional solid with non-interacting cracks will be derived. Also an approximate (but possibly exact) solution of crack-enhanced creep for dilute concentrations of penny-shaped non-interacting cracks will be given.

For both the two- and three-dimensional solid the cracks are assumed to be oriented such that the plane of the cracks is perpendicular to a uniaxially applied stress. Other stress states can be considered by the use of the appropriate expressions for the effect of cracks on elastic moduli [18–20]. For simplicity, for purposes of the analysis all cracks are assumed to be of uniform size. Variations in crack size can be taken into account by the use of appropriate distribution functions. In fact, as will be demonstrated, distributions in crack size need to be assumed to explain a number of creep phenomena in brittle ceramics. The absence of crack interactions will be assumed throughout. Finally, in order to fully concentrate on the effect of cracks on creep behaviour and to avoid a possible bias of the results obtained, no *a priori* choice of any specific mechanism of creep or crack growth will be made.

2.2. Two-dimensional model

2.2.1. Crack-enhanced creep

By regarding a crack to be composed of arrays of

dislocations, Weertman [15] derived the effect of cracks on the rate of creep as follows: viscous creep with the stress exponent $n = 1$, and dilute concentrations of non-interacting cracks:

$$\dot{\epsilon}_c = \dot{\epsilon}_0 (1 + 2\pi Na^2), \quad (1)$$

where $\dot{\epsilon}_c$ and $\dot{\epsilon}_0$ are the creep rates for the material with and without the cracks, respectively, N is the number of cracks per unit area and a is the crack half-length.

For power-law creep ($n > 1$) and dilute concentration of non-interacting cracks, an approximate expression for the creep rate is [15]:

$$\dot{\epsilon}_c = \dot{\epsilon}_0 (1 + 2\pi Na^2 n^{1/2}). \quad (2)$$

The total elastic creep strain which results over a given time period, can be obtained by the appropriate integration of Equations 1 or 2.

2.2.2. Elastic creep by crack growth

The rate of elastic creep due to crack growth can be derived as follows: Young's modulus of elasticity of the plate in the direction of the applied stress (perpendicular to the plane of the cracks) for conditions of plane stress is:

$$E = E_0 (1 + 2\pi Na^2)^{-1}, \quad (3)$$

where E and E_0 are Young's modulus of the plate with and without cracks, respectively.

The elastic strain in the direction of stress (σ) is:

$$\epsilon_e = \sigma/E = (1 + 2\pi Na^2)\sigma/E_0. \quad (4)$$

Differentiation of Equation 4 with respect to time yields the rate of creep in terms of the rate of crack growth (\dot{a}):

$$\dot{\epsilon}_e = 4\pi Na\dot{a}\sigma/E_0. \quad (5)$$

The total elastic creep strain, ϵ_e , which results over a time period, t , can be obtained by integration of Equation 5:

$$\epsilon_e = \int_0^t 4\pi\sigma Na(t)\dot{a}(t)/E_0 dt. \quad (6)$$

2.3. Three-dimensional model with penny-shaped cracks

2.3.1. Elastic creep by crack growth

Young's modulus of a solid with dilute concentration of oriented penny-shaped cracks, perpendicular to the plane of the cracks is [21]:

$$E = E_0 [1 + 16(1 - \nu_0^2)Na^3/3]^{-1}, \quad (7)$$

where ν_0 is Poisson's ratio of the crack-free solid.

In direct analogy to the derivation of Equation 5, the rate of elastic creep by growth of a penny-shaped crack, becomes:

$$\dot{\epsilon}_e = 16(1 - \nu_0^2)\sigma a^2 \dot{a}N/E_0. \quad (8)$$

Integration of Equation 8 over the appropriate time period, results in the total creep strain due to this mechanism.

2.3.2. Crack-enhanced creep

An approximate (but possibly exact) expression is suggested for the crack-enhanced creep for dilute concentration of non-interacting cracks. This suggested expression is based on the observation that, for the two-dimensional solid with through cracks, the role of the cracks in terms of the factor $(1 + 2\pi Na^2)$ is identical for the effective Young's modulus (Equation 3) as for the crack-enhanced creep (Equation 1). Of course, this is expected since the derivation for the crack-enhanced creep is based on the substitution of the viscosity for the shear modulus in the expression for the displacement [15]. Using the identical approach for the penny-shaped crack, with the aid of Equation 7 (with $\nu_0 = 0.5$ to maintain constant volume during creep) the crack-enhanced rate of viscous (linear) creep, becomes:

$$\dot{\epsilon}_c = \dot{\epsilon}_0 (1 + 4Na^3) \quad (n = 1). \quad (9)$$

For non-linear creep ($n > 1$) in direct analogy to Equation 2, the rate of crack-enhanced creep should be given approximately by:

$$\dot{\epsilon}_c = \dot{\epsilon}_0 (1 + 4Na^3 n^{1/2}). \quad (10)$$

3. Discussion

A number of aspects pertaining to the separate phenomena of elastic creep by crack growth and crack-enhanced creep, in terms of the crack-growth behaviour in polycrystalline ceramics will be discussed. The combined effects of these two creep phenomena are then discussed in terms of the effect of stress, grain size and other relevant variables on the kinetics of the creep process.

3.1. Nature of crack formation in polycrystalline ceramics at high temperature

Experimental evidence reported in a number of studies indicates that crack formation in polycrystalline ceramics takes the form of intergranular cracking along grain boundaries oriented more or

less perpendicularly to the applied tensile stress [6, 9, 11]. The total crack extension, at least during the initial stages of deformation, appears to be limited to the size of the grain-boundary facet. Most likely, pores located on the grain boundary or at triple points, constitute the precursor for such grain-boundary cracks. The growth of the cracks could occur by diffusional processes such as described by Rice, Chuang and co-workers [22–24] and others [25–30]. Once the crack has traversed the grain-boundary facet it is arrested, possibly due to the removal of the stress singularity, due to sliding on adjacent non-co-planar grain boundaries as discussed by Evans [31]. Thermodynamic considerations indicate that, at a given level of tensile stress, the precursor for a grain-boundary crack should have a minimum size, if it is to grow by a diffusional process. By analogy, a grain-boundary crack precursor of a given size will require a minimum value of stress for crack growth to occur. Crack precursors are not expected to be of uniform size but to exhibit a distribution of sizes. For this reason, the number (or density) of grain-boundary facets which undergo cracking is expected to increase with increasing stress level. At stresses sufficiently low that even the largest crack precursor cannot undergo growth, the crack density will be zero. At some intermediate stress level at which a fraction of the crack-precursors will grow into grain-boundary cracks, the crack density achieved will be denoted, N_σ . The maximum crack density, N_∞ is achieved when all crack precursors develop into grain-boundary cracks, which is expected to correspond to approximately one crack per grain. As will be shown later, the dependence of the crack density on stress, is critical for an explanation of the observed dependence of creep rate on stress and grain size.

At a given stress level, a crack precursor of sufficient size to undergo growth, initially will exhibit a crack velocity which increases with time. Maximum rate of crack growth will be reached at some intermediate position of the crack front on the grain-boundary facet. As the crack approaches the opposite side of the facet, the velocity will decrease with increasing crack extension, to reach zero value when the crack has completely traversed the grain-boundary facet. This crack growth behaviour is depicted schematically in Fig. 1.

For a given number of cavities per grain and a given amount of residual pore content, the size of the cavities will increase with grain size. As a

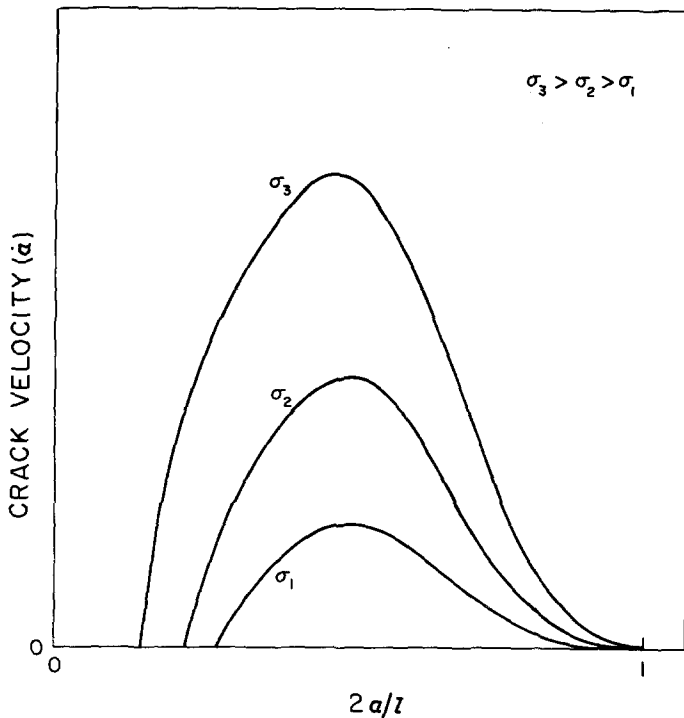


Figure 1 Schematic rate of crack growth as a function of relative position along grain-boundary facet, for three different stress values.

direct consequence, the stress level required for cavity extension is expected to decrease with increasing grain size. Furthermore, at a given stress level, large-grained materials are more likely to exhibit grain-boundary cracking than fine-grained materials. These latter observations will be shown to be relevant to the effect of grain size on creep rate. For purposes of further discussion, the existence of a one-to-one correspondence between the size of the crack precursors and grains will be assumed. Clearly, for the interpretation of creep data for any given material, such a correspondence must be established by metallographic or other techniques.

In polycrystalline ceramics, the size of grains and associated crack precursors generally is not uniform but will exhibit a size distribution. For this reason, higher concentrations of intergranular cracks are expected to be found in large-grained regions within the ceramic specimens than in regions with smaller grain sizes.

3.2. Elastic creep by crack growth

Elastic creep by crack growth does not require the simultaneous occurrence of creep by some other mechanism. For instance, subcritical crack growth was found to exist in polycrystalline aluminium oxide at temperatures as low as 600°C [32]. At this temperature, other mechanisms of

creep are not expected to make any significant contribution to the total creep deformation, so that elastic creep by crack growth is the dominant mechanism of creep. Non-linear deformation in heavily microcracked polycrystalline ceramics at room temperature was attributed to elastic creep by crack growth due to a stress-corrosion mechanism [33].

From the known values of the rate of crack growth, crack size and crack density at any instant in time, the corresponding rate of elastic creep can be calculated by means of Equation 5 or 8. In view of the crack-growth behaviour as a function of crack length shown in Fig. 1, the rate of elastic creep will show an initial increase, pass through a maximum followed by a decrease to zero as the cracks have grown across the grain-boundary facet. The creep strain as a function of time which results from this type of crack growth, is expected to be sigmoidal in nature and to reach a constant value when all cracks have reached their final size; as time approaches infinity. This time dependence of elastic strain is shown schematically in Fig. 2. The increase in elastic strain with increasing stress can be due to the increase in elastic strain at a given density of cracks as well as due to the increase in density of cracks with increasing stress.

An estimate can be made of the maximum strain which can result from elastic creep by crack

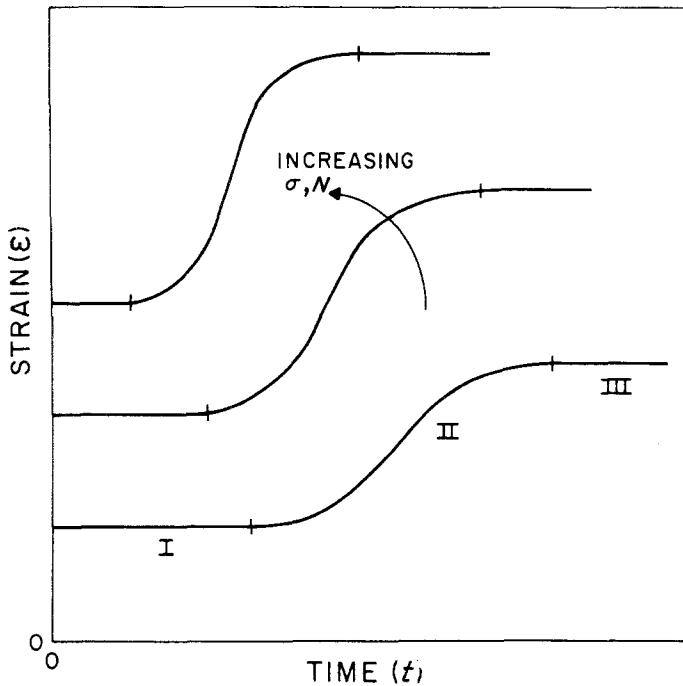


Figure 2 Elastic strain due to creep by crack growth, as a function of time of loading.

growth. The maximum crack density which can occur will correspond to one crack per grain. For the two-dimensional model let it be assumed first than the grains have a square geometry. In this case, the grain size $d = 2a$. Furthermore, for one crack per grain, the crack density $N = d^{-2}$ or $(4a^2)^{-1}$. Substitution of this latter quantity into Equation 5 suggests that for the present example, Young's modulus is reduced by a factor of 2.5. The total creep strain due to crack growth is then about 1.5 times the original elastic strain which results from the application of the load. The grain geometry more likely will be hexagonal so that, $a \approx d/4$. This latter value will result in a maximum creep strain by crack growth somewhat less than 0.5 of the original elastic strain. Similar values can be obtained for the penny-shaped cracks. In general then, depending on the shape of the grain, the maximum creep strain due to crack growth will be of the order of a small multiple of the original elastic strain. In typical creep experiments on polycrystalline ceramic materials the elastic strain at higher stress levels is of the order of 10^{-4} , with a corresponding elastic creep strain of the same value. Total creep strains obtained over long durations can be of the order of a few percent. For this reason, if other creep mechanisms also are operative, elastic creep by crack growth can make a contribution which is a small fraction of the total creep strain. If, however, other creep mech-

anisms are virtually absent, elastic creep by crack growth is the only (i.e. dominant) mechanism of creep. This latter condition is expected to arise at low temperatures, at which crack growth takes place by stress corrosion or by the flow of a low viscosity grain-boundary phase [31]. This latter mechanism was concluded to be responsible for the residual stress relaxation in a polycrystalline aluminium oxide at temperatures as low as 850°C [17].

3.3. Crack-enhanced creep

The form of Equations 1, 2, 9 and 10 indicates that the presence of the cracks causes an increase in the rate of creep over the creep rate in the absence of cracks, by a factor which depends on the absence or presence of crack interaction, the stress dependence of the creep mechanism and the crack geometry. It is of importance to note that the expressions for the effective Young's modulus and crack-enhanced creep for dilute concentrations of cracks for the two-dimensional model contain the identical factor $(1 + 2\pi Na^2)$.

This indicates that the relative increase in elastic strain due to creep by crack growth and the relative increase in the rate of the basic creep are numerically identical. As indicated earlier the creep strain which results from the growth of cracks is of the order of a multiple (2 to 4) of the original elastic strain attained on application of the

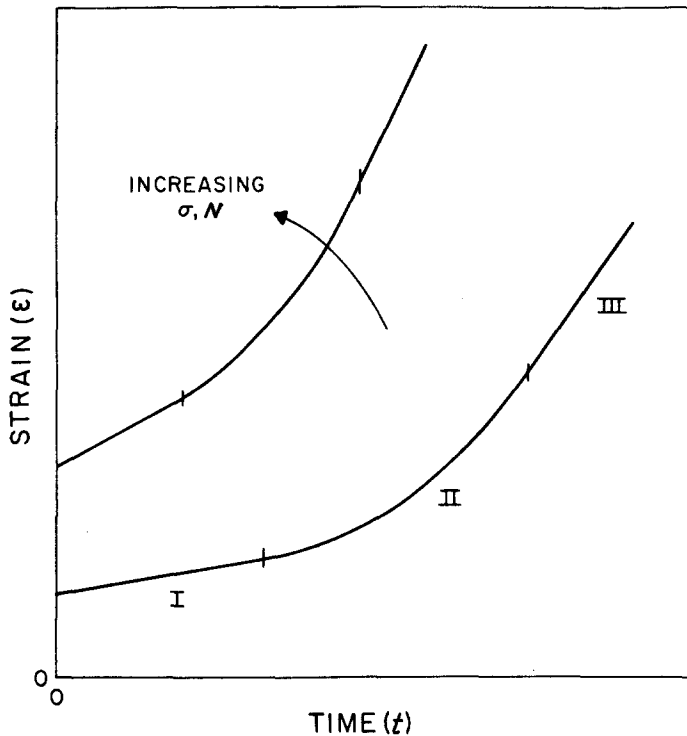


Figure 3 Crack-enhanced creep strain as a function of time of loading.

load. The increase in the basic creep rate will be equal to the same amount, which must be considered a significant effect. In view of the form of Equations 2 and 10, this effect is even larger for power-law creep.

Fig. 3 shows the creep curve for crack-enhanced creep for a given value of stress, σ . In region I, no cracks have developed. In region II, crack growth occurs, which results in a gradual increase in slope with an instantaneous value of slope $\dot{\epsilon}_0(1 + 2\pi N_\sigma a^2)$ and curvature of $\ddot{\epsilon} = 4\pi\dot{\epsilon}_0 N_\sigma \dot{a} a$, where a and \dot{a} represent instantaneous values. Region III represents the regime of crack-enhanced creep in which the cracks have reached their final length, a_∞ .

3.4. The creep curve

Elastic creep by crack growth and crack-enhanced creep are expected to occur concurrently. For this reason, the creep curve for a polycrystalline ceramic undergoing crack formation can be obtained by superposition of both effects, shown schematically in Fig. 4. A total of four creep regions can be identified. In region I, the material creeps at a rate which corresponds to the absence of cracks. In region II, elastic creep is dominant with a superimposed contribution of crack-enhanced creep, at a rate which corresponds to the instantaneous value of crack size, a_1 , and

crack density, N_σ . Region III represents the steady-state crack-enhanced creep for constant crack size, a_∞ , and density, N_σ . Note that the creep strain in region III represents the sum of the crack-enhanced creep strain and the elastic creep strain achieved in Region II. Region IV, in which crack-coalescence occurs, includes additional elastic as well as crack-enhanced creep, leading to inevitable failure. A detailed analysis of such effects is beyond the scope of this study.

Fig. 4 includes three curves corresponding to three levels of stress. At the lowest stress level, crack formation will not occur, so that the ceramic will exhibit stage I creep only. At the intermediate stress level, cracks will form by the growth of a fraction of the crack precursors to result in a crack density N_σ . The creep curve for the highest stress level corresponds to the maximum possible crack density, N_∞ .

The decrease in slope as creep progresses from stage II to III may explain the frequently observed decrease in creep rate with increasing time, as for instance reported by Coble [39] and Evans [34]. Since elastic creep by crack growth can commence immediately on application of the load and continue over the total duration of creep, stage I and II as well as II and III are expected to merge, without clear demarcation.

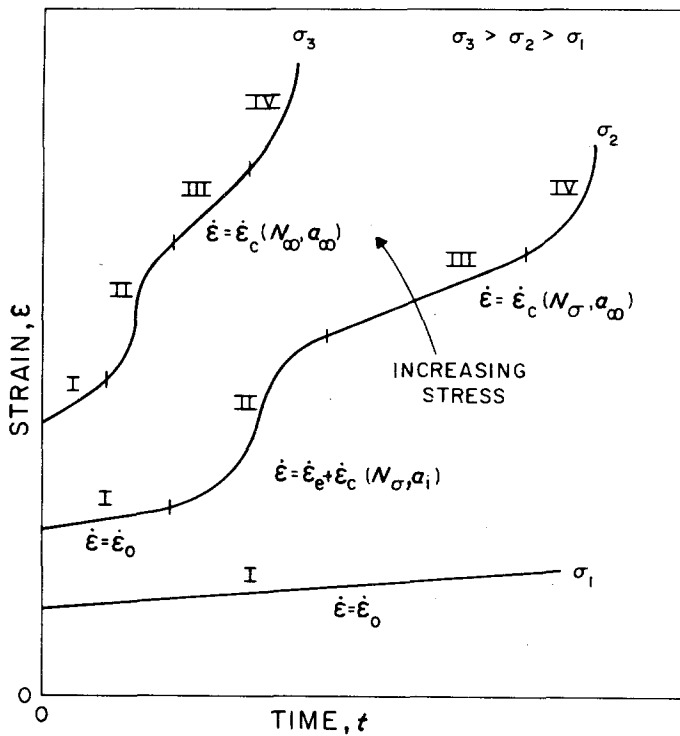


Figure 4 Combined contribution of elastic creep by crack growth and crack-enhanced creep, to creep deformation of polycrystalline ceramic, for three stress values.

3.5. Stress dependence of steady-state creep

The stress exponent of a creep mechanism can be obtained by measuring the slope of a log-log plot of the creep rate corresponding to a number of stress values. It is of interest to speculate on the slope of such a plot in the light of the present discussion. For simplicity, steady-state creep in stages I and III will be considered, so that the size and density of cracks at any stress value are fully developed. At sufficiently low values of stress, at which no crack formation occurs, the slope of the log $\dot{\epsilon}$ -log σ curve will correspond to the stress exponent of the underlying creep mechanism. At higher stresses, the change in slope will depend critically on the distribution function of the size of the precursors from which the grain-boundary cracks form, as shown schematically in Fig. 5. For crack precursors of identical size, the log $\dot{\epsilon}$ -log σ plot is expected to show a discontinuity at the minimum level of stress required for crack growth to occur. The magnitude of this discontinuity is equal to the factor $(1 + 2\pi N_{\infty} a^2)$. Both above and below this discontinuity, the slopes of the log $\dot{\epsilon}$ -log σ plot will be identical, corresponding to the stress exponent of the basic mechanism of creep.

For a distribution of sizes of crack precursors,

as discussed earlier, the crack density becomes a function of the applied stress. In this case, rather than showing a discontinuity, the log $\dot{\epsilon}$ -log σ plot is expected to exhibit a more continuous sigmoidal behaviour as indicated in Fig. 5. This latter situation is expected to be the case in practice. Experimental data generally show an increase in slope with increasing stress [6-11]. The present writers, however, are not aware of experimental data for decrease in slope with increasing stress at the highest values of stress, as suggested by the present discussion. Quite possibly, in practice, this effect is hidden by the presence of stage IV creep. Nevertheless, it could be observed for materials in which crack coalescence, for whatever reason, is suppressed.

A numerical value for the slope, i.e. the apparent stress exponent of the log $\dot{\epsilon}$ -log σ plot, can be derived. Let it be assumed, quite arbitrarily, that the density of cracks in steady-state stage III creep, as a function of stress, can be expressed by a Weibull distribution of the form:

$$N/N_0 = 1 - \exp[-(\sigma/\sigma_0)^m], \quad (11)$$

where N_0 , σ_0 and m are constants.

Let us take the condition that the stress value (σ) is such that:

$$(\sigma/\sigma_0)^m \ll 1, \quad (12)$$

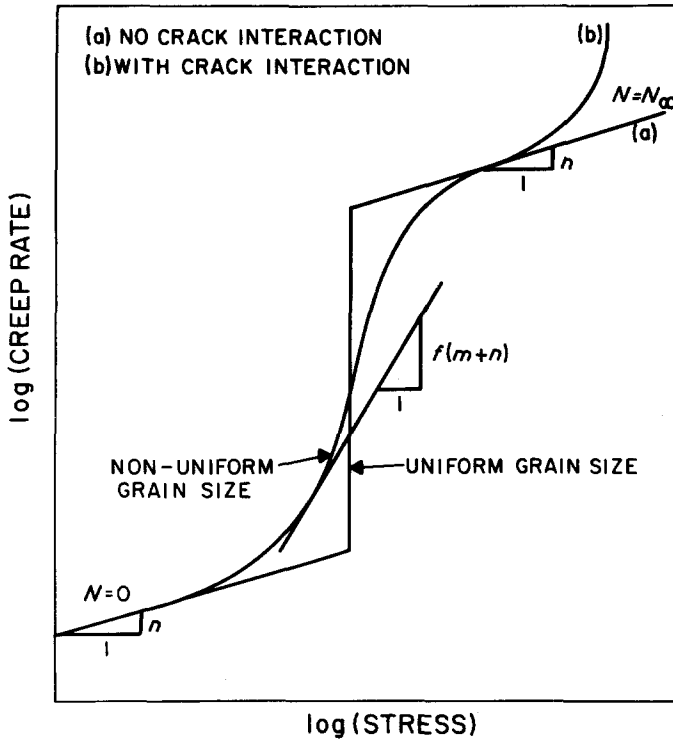


Figure 5 Stress dependence of rate of steady-state crack-enhanced creep of polycrystalline ceramic, for uniform and non-uniform grain-size distributions and a fixed ratio of the size of crack precursor to grain size.

which yields:

$$N/N_0 \approx (\sigma/\sigma_0)^m. \quad (13)$$

The incremental increase in creep rate in stage III creep over stage I creep for viscous or power-law creep, from Equations 1 and 2 can be written:

$$\dot{\epsilon}_c - \dot{\epsilon}_0 = 2\pi\dot{\epsilon}_0 N a^2 n^{1/2}. \quad (14)$$

Setting $\dot{\epsilon}_0$ in the right-hand side of Equation 14 equal to $\dot{\epsilon}_0 = A\sigma^n$ and taking logarithms yields:

$$\begin{aligned} \log(\dot{\epsilon}_c - \dot{\epsilon}_0) &= (n + m) \log \sigma \\ &+ \log(2\pi A N_0 \sigma_0^{-m} a^2 n^{1/2}). \end{aligned} \quad (15)$$

This result (Equation 15) shows that the slope of the plot of the log of the incremental increase in creep rate against $\log \sigma$ has a value equal to $n + m$. In other words, the stress exponent observed in steady-state creep is a function of both the stress exponent of the creep mechanism as well as that of the crack distribution.

For other types of distribution functions undoubtedly other relations for the apparent stress exponent can be obtained. Qualitatively, however, regardless of the formulation used, it is expected that such stress exponents will be found to depend on the density of cracks as a function of stress. Critical to note is that the formation of

cracks during creep can cause a ceramic to exhibit power-law creep, even when the underlying creep mechanism is of a viscous (linear) nature.

3.6. Effect of grain size on rate of creep

The grain-size exponent for the dominant mechanism of creep can be obtained from the slope of the log-log plot of the creep rate at a given value of stress, against grain size. For pure Nabarro-Herring or Coble creep, the grain-size exponents are -2 and -3 , respectively. The effect of cracks on the apparent grain-size exponent will be examined. For simplicity, it will be assumed that the ratio of the size of the precursor crack to the size of the grains is a constant for either a uniform grain size or a distribution of grain sizes. In this manner, for any given stress level, the minimum size of precursor required for grain-boundary cracks to form, can be expressed directly in terms of the grain size. As stated earlier, the exact relationship between precursor size and grain size, for any given material must be established by metallographic means. Also, the discussion will focus on steady-state crack-enhanced creep, i.e. stages I and III only.

Fig. 6 indicates schematically the effect of the presence of the cracks on the dependence of creep rate on grain size. For grain sizes and

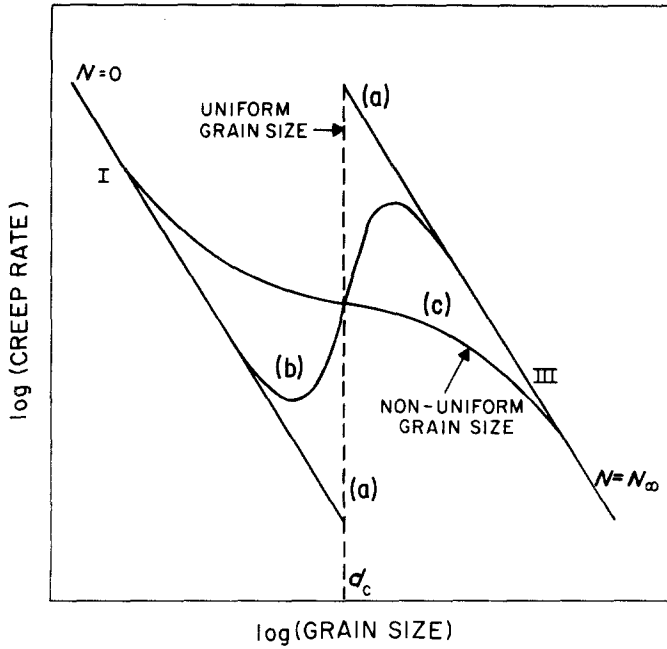


Figure 6 Grain-size dependence of steady-state crack-enhanced creep, for fixed ratio of the size of crack-precursor to grain size. (a) Uniform grain size; (b) narrow grain-size distribution; (c) broad grain-size distribution.

associated crack precursors which are too small for crack formation to occur, the creep rate as a function of grain size is depicted in the left-hand part of the diagram, denoted by $N = 0$. Let it first be assumed that the grain size (and precursor size) is uniform. At any given stress level, this implies that grain-boundary cracking will occur at and above a critical value of grain size. On the log-log plot of creep rate against grain size, this will result in a discontinuity equal to the factor $(1 + 2\pi N_\infty a_c^2)$ as indicated in Fig. 6. Above and below this discontinuity the slope of the $\log \dot{\epsilon} - \log d$ plot should be identical and equal to the grain-size exponent of the basic creep mechanism. The other two curves in Fig. 6 indicate the grain-size dependence of the creep rate for narrow and broad grain-size distributions. The narrow grain-size distribution shows an increase in creep rate with increasing grain size, near the value of critical size for the uniform grain size. For the broad grain-size distribution, the creep rate decreases monotonically with increasing grain size, however, with a slope near the value of critical grain size, less than the grain-size exponent for the underlying mechanism of creep.

A value for the apparent grain-size exponent as affected by the presence of cracks can be derived in a manner analogous to the derivation for the apparent stress exponent, obtained in Section 3.5.

Again, let it be assumed that the density of cracks can be expressed by a Weibull function:

$$N/N_0 = 1 - \exp[-(d/d_0)^m], \quad (16)$$

where N_0 and d_0 are constants. Assuming that

$$(d/d_0)^m \ll 1 \quad (17)$$

yields:

$$N/N_0 \simeq (d/d_0)^m. \quad (18)$$

The rate of crack-enhanced creep (Equation 1) can be written in terms of the grain size as:

$$\dot{\epsilon}_c = A' d^b (1 + 2\pi N a^2), \quad (19)$$

where A' is a constant which depends on stress and temperature. It will be recalled that for intergranular cracks, the crack dimension is a function of the grain size. Assuming the grain shape to be hexagonal, the crack size $a \simeq d/4$, which upon substitution into Equation 19 yields:

$$\dot{\epsilon}_c = A' d^b (1 + \pi N d^2 / 8). \quad (20)$$

Again it is convenient to consider the incremental increase in creep rate due to the cracks, which with the aid of Equation 18 can be expressed:

$$\dot{\epsilon}_c - \dot{\epsilon}_0 = \pi A' N_0 d^{(b+m+2)} / 8 d_0^m. \quad (21)$$

Taking logarithms yields:

$$\begin{aligned} \log(\dot{\epsilon}_c - \dot{\epsilon}_0) &= (b + m + 2) \log d \\ &+ \log(\pi A' N_0 / 8 d_0^m). \end{aligned} \quad (22)$$

Equation 22 shows that a plot of the log of the incremental increase in creep rate due to the cracks against the logarithm of the grain size, will suggest an apparent grain-size exponent equal to $b + m + 2$.

It is suggested that this effect could offer an explanation for the observation of Burton *et al.* [35] on the grain-size dependence of the creep behaviour of polycrystalline UO_2 .

The effect of non-uniformity in grain-size distribution on creep and creep-fracture behaviour has received recent attention. Raj and Ghosh [36] and Schneibel *et al.* [37] analysed the creep behaviour of polycrystalline materials with a distribution in grain sizes. Owing to the higher creep rate in the fine-grained material, the coarse grains, due to the stress transfer, became subjected to a higher stress value, which could make them undergo power-law creep. Evans [38] attributed the effect of stress transfer to the premature failure of polycrystalline ceramics in creep fracture due to the existence of clusters of coarse-grained material within a fine-grained matrix. It is conceivable that the presence of cracks in the coarse-grained material can modify the above effects. At the proper combinations of stress, grain-size distributions and size of crack precursors, the coarse-grained material due to the combined effects of stress-enhanced creep, as discussed above, and elastic creep, can exhibit creep rates well above those for the fine-grained material. If so, the stress transfer will occur from the coarse to the fine grains with corresponding changes in creep kinetics and creep-fracture behaviour. Space considerations require deferring detailed analysis of these effects to a future study.

3.7. Activation energy

The presence of the cracks is also expected to have an effect on the value of the activation energy which can be inferred from a plot of the creep rate at a given stress, against the reciprocal of the absolute temperature. For stage I and III steady-state creep, in which crack growth is absent, the activation energy should be identical to the activation energy for the creep rate, $\dot{\epsilon}_0$, for the underlying mechanism of creep. For stage II creep, however, the observed rate of creep is the sum of the instantaneous values of the elastic creep rate due to crack growth and the crack-enhanced creep. In this case, the observed activation energy is expected to be a function of the

activation energy for the basic creep mechanism as well as the activation energy for crack growth. These latter two quantities are not necessarily equal. In particular, large differences could be found in these values, if the basic creep mechanism were controlled by volume diffusion as in the case of Nabarro–Herring creep, with crack growth governed by surface and grain-boundary diffusion.

The effect of such differences on the creep kinetics is shown schematically in Fig. 7. The lower and upper curves correspond to stage I and III creep for the absence of cracks ($N = 0$) and a fully developed crack system ($N = N_\sigma$, $a = a_\infty$), respectively. These lines have identical slopes corresponding to the activation energy for the creep rate, $\dot{\epsilon}_0$. The separate contribution of elastic creep by crack growth and the crack-enhanced creep in stage II, for an instantaneous value of crack size (a_i) are indicated by the curves drawn intermediate to the stages I and III creep. For purposes of illustration, the creep rates for these two separate mechanisms were taken as approximately equal. The curve for the crack-enhanced creep $\dot{\epsilon}_c(N_\sigma, a_i)$ is parallel to the curves for stages I and III creep. The curve for the elastic creep by crack growth, however, exhibits a lower slope because of the lower value for the activation energy for crack growth than the corresponding value for the creep rate, $\dot{\epsilon}_0$. Fig. 7 also includes the sum of the stage II elastic and crack-enhanced creep. As shown, the slope of this curve is no longer constant, but has a lower value of activation energy at the lower temperature and a higher value at the higher temperatures. Furthermore, as creep advances, a decrease in activation energy should occur in going from stage I to stage II, followed by an increase as creep advances from stage II to stage III creep. This latter effect possibly may offer an explanation for the findings of Crosby and Evans [9] that in polycrystalline Ni-doped Al_2O_3 , the presence of the cracks causes an increase in activation energy.

3.8. Recommendations

It can be concluded that cracks can have a significant effect on the creep behaviour of polycrystalline ceramics. An analysis of this effect and the interpretation of creep data for any given material, will require the generation of experimental data for the crack density and distribution of crack sizes. A measurement of the change in elastic

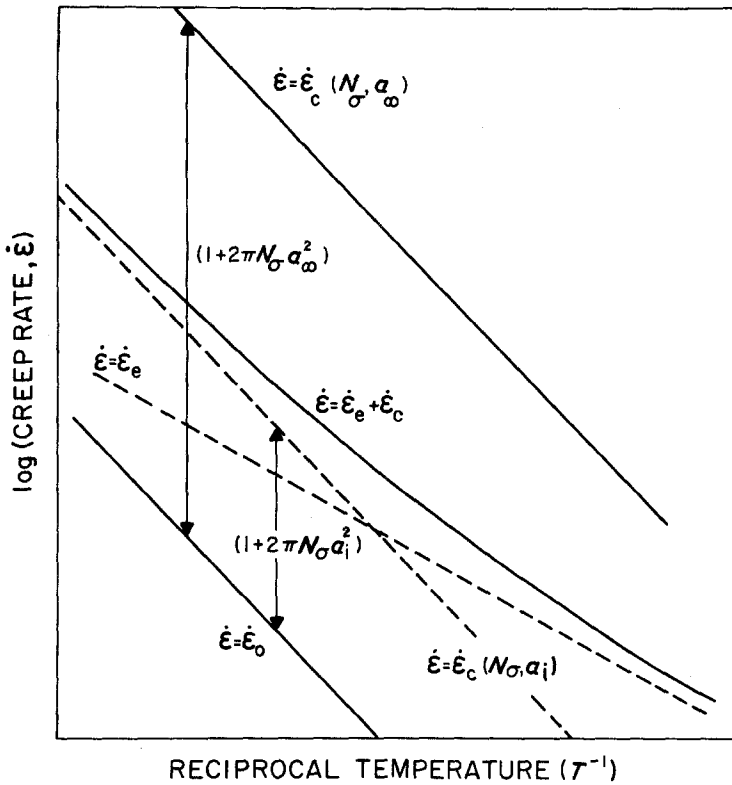


Figure 7 Arrhenius plot of rate of creep in regions I, II and III; indicating the effect of crack growth on the apparent activation energy.

properties as creep progresses should prove to be most helpful, as well. At higher temperatures, such a measurement on experimental grounds may prove to be difficult. As an alternative, it may suffice to measure the elastic properties of the samples before and after the creep experiment. A comparison of the stress-strain curve on loading and unloading, could give a valuable indication of the formation of the cracks and their role during creep. A separate measurement of elastic properties at room temperature by an appropriate method before and after the measurement of creep at higher temperatures should give accurate numerical data. Such measurements may prove too complex in specimens subjected to creep in bending, a method frequently employed to examine tensile creep behaviour in ceramics. Because of the non-uniform distribution of stress under those conditions, the cracks will not be distributed uniformly throughout the specimen, but are more likely to be found in the regions of the specimen at or near the maximum values of tensile stress. Inevitably, this causes the creation of spatially non-uniformly distributed elastic behaviour and an associated shift in the position of the neutral axis towards the regions in the specimen subjected to compression. Because of this latter effect, the quan-

titative interpretation of creep data in bending may prove to be more complex than in pure tension. Finite element modelling of such creep behaviour based on spatially and time-dependent elastic behaviour could be of value in the interpretation of the experimental data. If the thoughts expressed in this paper are found to be of merit, a structural analysis of the cracks formed should be a vital part of any study of the creep behaviour of polycrystalline ceramic materials.

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